

Finite Automata

- ◆ A finite automaton has a **set of states** and its **control** moves from state to state in response to external **inputs**
- ◆ **Deterministic: automation cannot be in more than one state at any time**
- ◆ **Nondeterministic: automation can be several states at once**

Informal Picture of Finite Automata

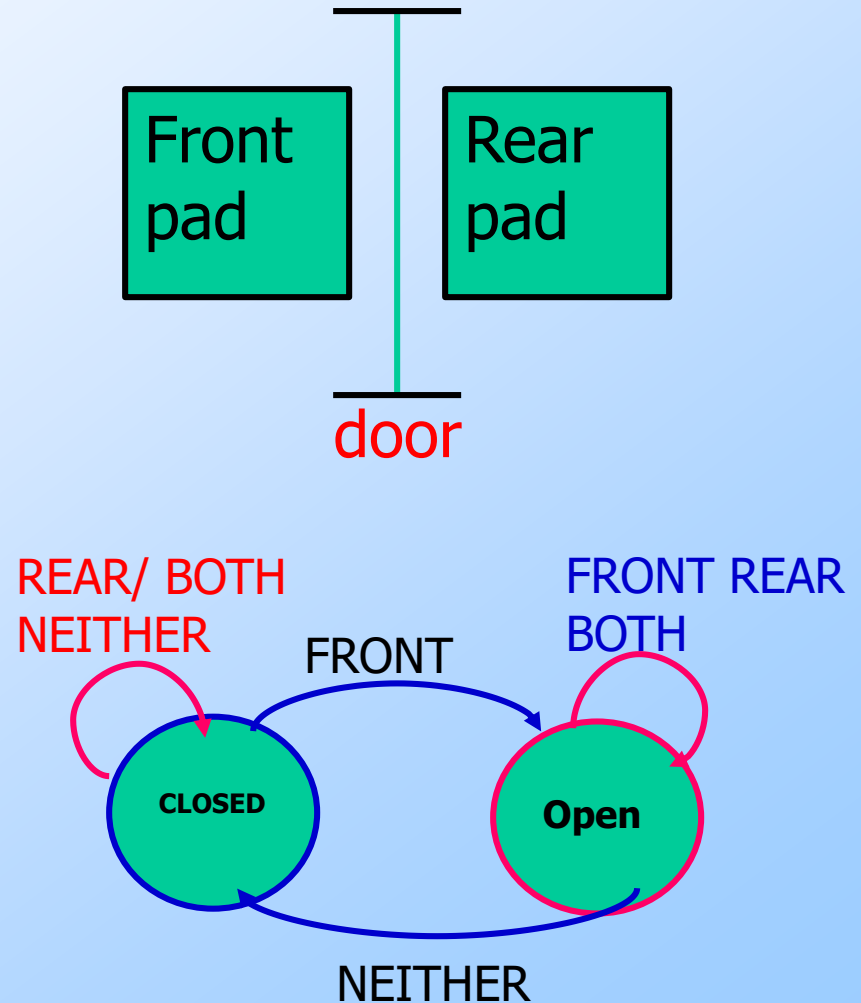
- ◆ Finite automata are good models for computers with an extremely limited amount of memory
 - electromechanical devices
- ◆ The controller for an automatic door is one example of such a device
- ◆ Often found in supermarket entrances/exits, automatic doors swing open when sensing that a person is approaching

Informal Picture of Finite Automata

◆ **Two states:** OPEN or CLOSED

◆ **04 input conditions:**

1. **FRONT** (a person is standing on the pad in front of the doorway)
2. **REAR** (a person is standing on the pad to the rear of the doorway)
3. **BOTH** (people are standing on both pads)
4. **NEITHER** (no one is standing on either pad)



State table

	Input signal			
	NEITHER	FRONT	REAR	BOTH
CLOSED	CLOSED	OPEN	CLOSED	CLOSED
OPEN	CLOSED	OPEN	OPEN	OPEN

- Controller moves from state to state, depending on input
- when CLOSED state & input NEITHER/REAR, it remains in CLOSED state
- If the input BOTH, it stays CLOSED because opening the door Risks knocking someone over on the rear pad
- But if input FRONT, it moves to the OPEN state
- In the OPEN state, if input FRONT/REAR/BOTH, it remains OPEN

Other Examples

- ◆ This controller is a computer that has just a single bit of memory, capable of recording which is two states the controller is in
- ◆ Others: elevator controller, dishwashers, thermostat, digital watches, calculators
etc

Deterministic Finite Automata (DFA)

- ◆ **Deterministic**: on each input there is one state to which the automaton can transition from its current state
- ◆ In terms of 5 tuples:

$$A = (Q, \Sigma, \delta, q_0, F)$$

- ◆ **A**: DFA, **Q**: set of states, **Σ** : input symbols, **δ** : transition function, **q_0** : start state, **F**: set of accepting state

DFA

- ◆ A formalism for defining languages, consisting of:
 1. A finite set of *states* (Q , typically).
 2. An *input alphabet* (Σ , typically).
 3. A *transition function* (δ , typically).
 4. A *start state* (q_0 , in Q , typically).
 5. A set of *final states* ($F \subseteq Q$, typically).
 - ◆ “Final” and “accepting” are synonyms denoted by **Double Loops**.

The Transition Function

- ◆ Takes two arguments: a state & an input symbol; returns a state
- ◆ δ represented by an arc between states and the labels on the arcs.
- ◆ $\delta(q, a)$ = the state that the DFA goes to when it is in state q & input a is received.
- ◆ q state (current) to p state (next): an arc labeled with a

How a DFA Process Strings

- ◆ How the DFA decides whether or not to 'accept' a sequence of input symbols
- ◆ The language of the DFA is the set of all strings that the DFA accepts.
- ◆ Sequence of input symbols: a_1, a_2, \dots, a_n
- ◆ initial state: q_0
- ◆ Transition function, $\delta(q_0, a_1) = q_1$ to find state that the DFA A enters after processing the first input symbol a_1

How a DFA Process Strings

- ◆ Process next input symbol, a_2 : $\delta(q_1, a_2) = q_2$
- ◆ continue in this way, finding states q_3, q_4, \dots, q_n , such that $\delta(q_{i-1}, a_i) = q_i$ for each i
- ◆ If q_n is a member of F , then the input $a_1 a_2 \dots a_n$ is **accepted**,
- ◆ if not then it is **rejected**.

Example

- ◆ Specify a DFA that accepts all and only the strings of 0's and 1's that have the sequence 01 somewhere in the string.
- ◆ L as:
 $\{w \mid w \text{ is of the form } x01y \text{ for some strings } x \text{ \& } y \text{ consisting of 0's \& } 1\text{'s only}\}$
- $\{x01y \mid x \text{ \& } y \text{ are any strings of 0's \& } 1\text{'s}\}$

Example

- ◆ Strings: 01, 11010, 100011 exists in L
- ◆ ϵ , 0, & 111000 not exist in L
- ◆ $\Sigma = \{0, 1\}$
- ◆ $\delta(q_0, 1) = q_0$
- ◆ $\delta(q_0, 0) = q_2$
- ◆ $\delta(q_2, 0) = q_2$
- ◆ $\delta(q_2, 1) = q_1$

Example

- ◆ $\delta(q_1, 0) = \delta(q_1, 1) = q_1$
- ◆ $Q = \{q_0, q_1, q_2\}, F = \{q_1\}$
- ◆ The complete specification of the automation A that accepts the L of strings that have a 01 substring:

$$A = (\{q_0, q_1, q_2\}, (0, 1), \delta, q_0, \{q_1\})$$

Notations for DFA's

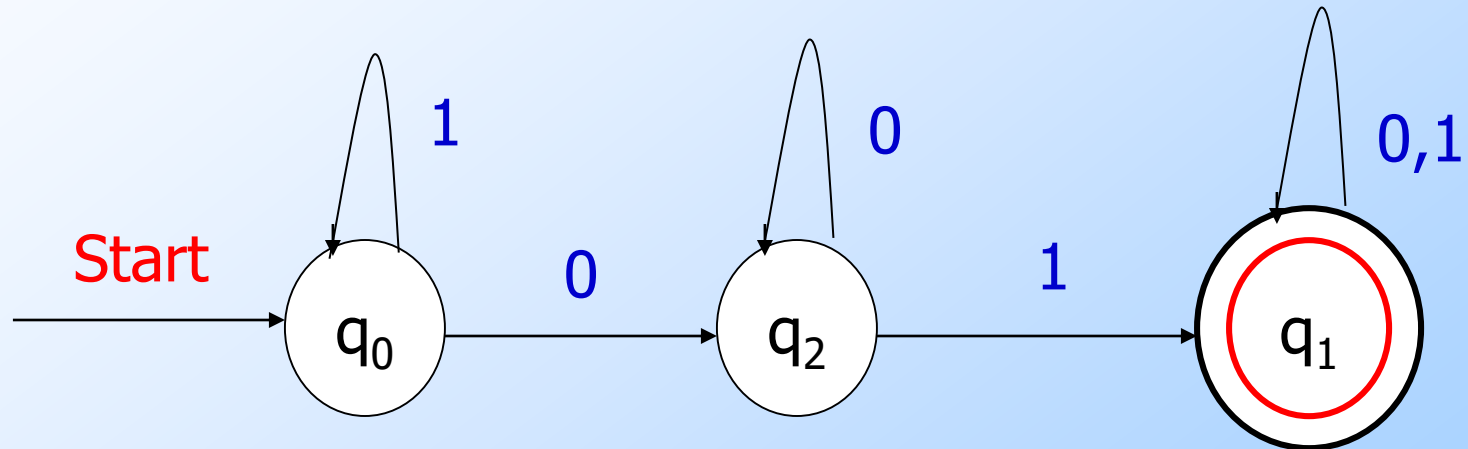
- ◆ Specifying a DFA as 5tuple with a detailed description of the δ transition function: **tedious & hard to read**
- ◆ Two notations:
 1. Transition diagram
 2. Transition table

Transition Diagram

- ◆ Nodes = states.
- ◆ Arcs represent transition function.
 - ◆ Arc from state p to state q labeled by all those input symbols that have transitions from p to q .
- ◆ Arrow labeled "Start" to the start state.
- ◆ Final states indicated by double circles.

Example

- ◆ Draw the Transition Diagram for the DFA accepting all string with a **substring 01**.

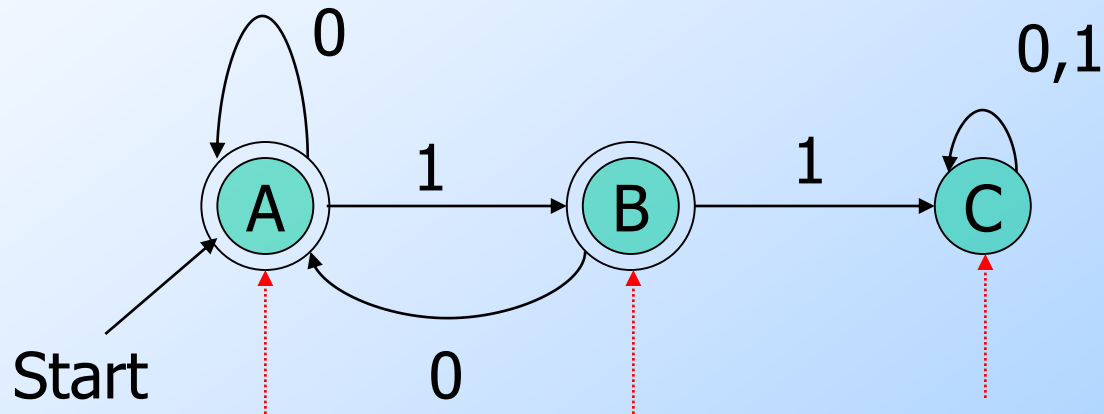


$$A = (\{q_0, q_1, q_2\}, \{0, 1\}, \delta, q_0, \{q_1\})$$

Check with the string 01, 11010, 100011,
0111, 110101, 11101101, **111000**

Another Example

Accepts all strings without two consecutive 1's.



Previous string OK, does not end in 1.

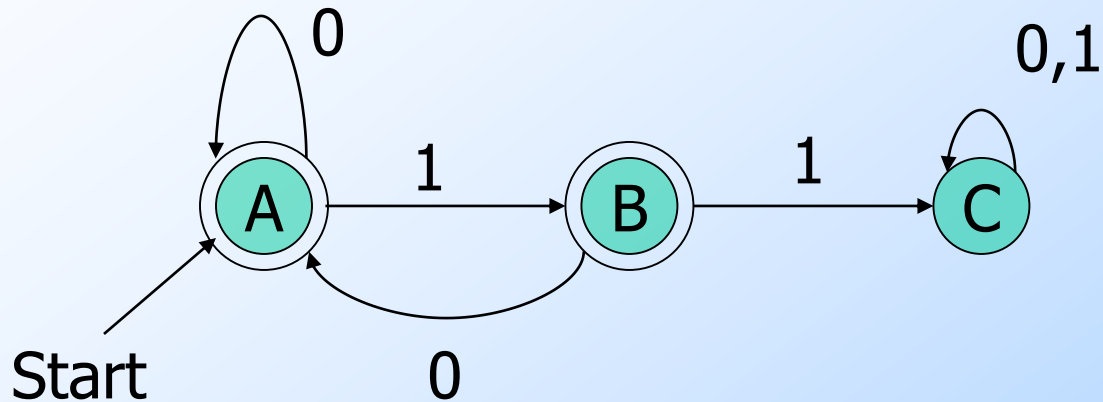
Previous String OK, ends in a single 1.

Consecutive 1's have been seen.

Transition Table

- ◆ a conventional, tabular representation of a function like δ that takes two arguments & return a value.
- **Rows:** states; corresponding to state q
- **Columns:** inputs; corresponding to input a is the state $\delta(q,a)$
- The start state is marked with an **arrow** and accepting states are marked with a **star**.

Example



Final states
starred

Columns =
input symbols

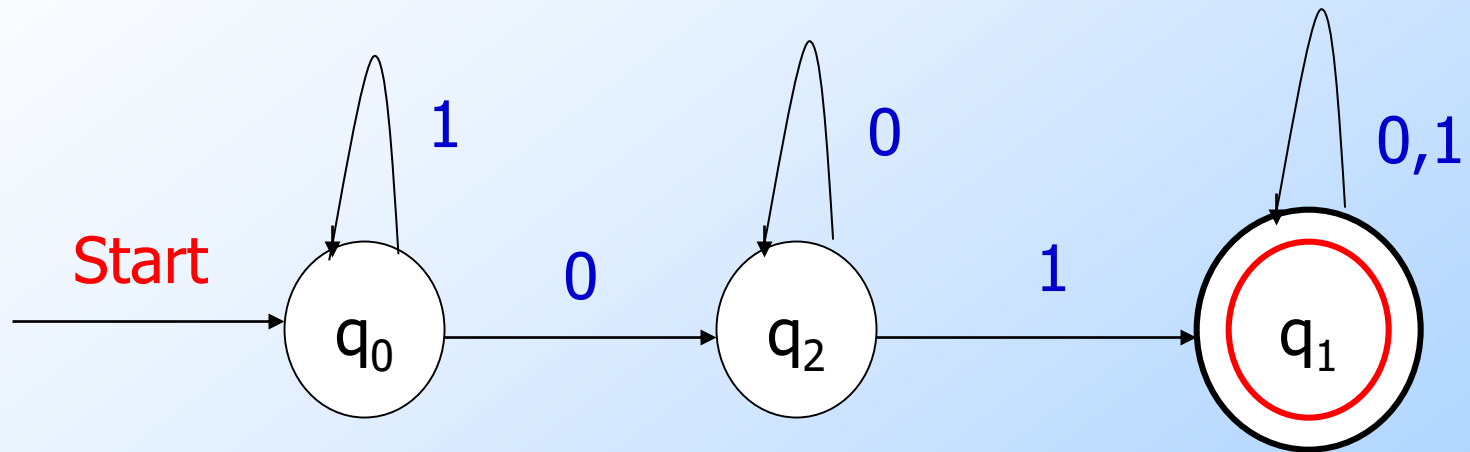
Arrow for
start state

→ * A
* B
C
↑

	0	1
A	A	B
B	A	C
C	C	C

Rows = states

Example



- ◆ $(q_0, 0) = q_2$
- ◆ $(q_0, 1) = q_0$
- ◆ $(q_1, 0) = q_1$
- ◆ $(q_1, 1) = q_1$
- ◆ $(q_2, 0) = q_2$
- ◆ $(q_2, 1) = q_1$

	0	1
$\rightarrow q_0$	q_2	q_0
$*q_1$	q_1	q_1
q_2	q_2	q_1

Extended Transition Function

- ◆ An Extended Transition Function that describes what happens when we start in any state and follow any sequence of inputs.

$\delta \rightarrow$ Transition Function

$\hat{\delta} \rightarrow$ Extended Transition Function

- a function that takes a state q & a string w & returns a state p -the state that the automation reaches when starting in state q & processing the sequence of inputs w

Extended Transition Function

- ◆ We describe the effect of a string of inputs on a DFA by extending δ to a state and a string.
- ◆ Induction on length of string.
- ◆ **Basis:** $\hat{\delta}(q, \epsilon) = q$
- ◆ **Induction:** $\hat{\delta}(q, wa) = \delta(\hat{\delta}(q, w), a)$
 - ◆ w is a string; a is an input symbol.

Induction

$$2. \hat{\delta}(q, w) = \delta(\hat{\delta}(q, x), a) = \delta(p, a)$$

$w \rightarrow xa$

$a \rightarrow$ last symbol = 1

$w \rightarrow$ string = 1101

$x \rightarrow$ string consisting of all but the last symbol = 110

Extended δ : Intuition

◆ Convention:

- ◆ ... w, x, y, x are strings.
- ◆ a, b, c, \dots are single symbols.

- ◆ Extended δ is computed for state q and inputs $a_1 a_2 \dots a_n$ by following a path in the transition graph, starting at q and selecting the arcs with labels a_1, a_2, \dots, a_n in turn.

Example: Extended Delta

	0	1
A	A	B
B	A	C
C	C	C

$$\delta(B,011) = \delta(\delta(B,01),1) = \delta(\delta(\delta(B,0),1),1) =$$
$$\delta(\delta(A,1),1) = \delta(B,1) = C$$

Delta-hat

- ◆ In book, the extended δ has a "hat" to distinguish it from δ itself.
- ◆ Not needed, because both agree when the string is a single symbol.
- ◆ $\delta(q, a) = \hat{\delta}(\delta(q, \epsilon), a) = \delta(q, a)$

Extended deltas



Example

◆ Let us design a DFA to accept the language

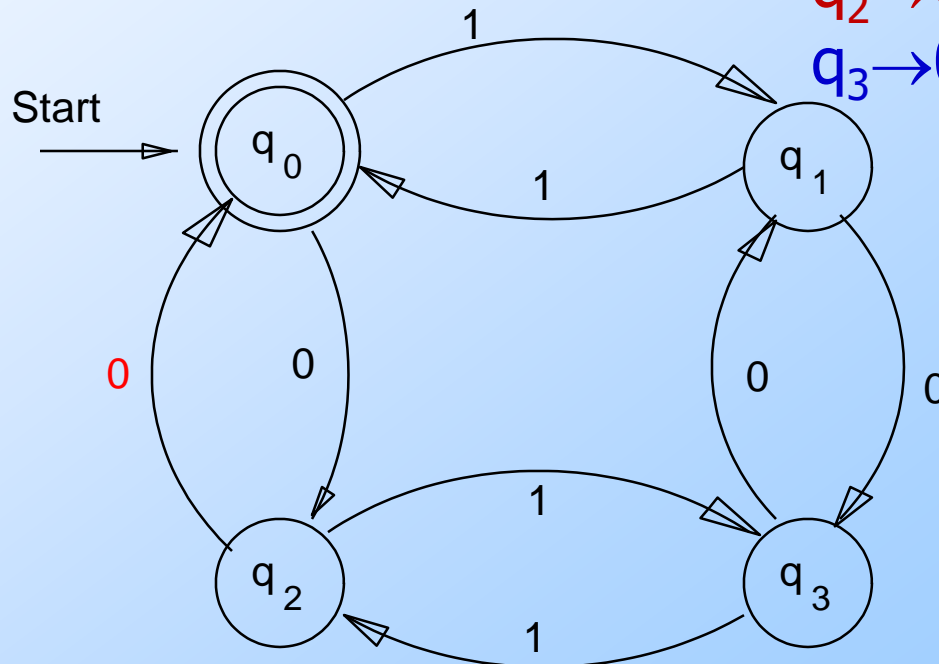
$L = \{w \mid w \text{ has both an even number of 0's and even number of 1's}\}$

$q_0 \rightarrow 0(\text{even}) \ 1(\text{even})$

$q_1 \rightarrow 0(\text{even}) \ 1(\text{odd})$

$q_2 \rightarrow 0(\text{odd}) \ 1(\text{even})$

$q_3 \rightarrow 0(\text{odd}) \ 1(\text{odd})$



	0	1
$\rightarrow^* q_0$	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_3	q_1

Example with Transition Table

	0	1
$\rightarrow^* q_0$	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_3	q_2

◆ $w = 110101$

◆ $x = 11010$

◆ $a = 1$

$\hat{\delta}(q_0, 110101) = q_0$

- $\hat{\delta}(q_0, \epsilon) = q_0$
- $\hat{\delta}(q_0, 1) = \delta(\hat{\delta}(q_0, \epsilon), 1) = \delta(q_0, 1) = q_1$
- $\hat{\delta}(q_0, 11) = \delta(\hat{\delta}(q_0, 1), 1) = \delta(q_1, 1) = q_0$
- $\hat{\delta}(q_0, 110) = \delta(\hat{\delta}(q_0, 11), 0) = \delta(q_0, 0) = q_2$
- $\hat{\delta}(q_0, 1101) = \delta(\hat{\delta}(q_0, 110), 1) = \delta(q_2, 1) = q_3$
- $\hat{\delta}(q_0, 11010) = \delta(\hat{\delta}(q_0, 1101), 0) = \delta(q_3, 0) = q_1$
- $\hat{\delta}(q_0, 110101) = \delta(\hat{\delta}(q_0, 11010), 1) = \delta(q_1, 1) = q_0$

Accepted

Example: Try Yourself

◆ $A = \{w \mid w \text{ contains at least one } 1 \text{ and an even number of } 0\text{s follow the last } 1\}$

◆ **Hints:** $A_1 = (Q, \Sigma, \delta, q_1, F)$

1. $Q = \{q_1, q_2, q_3\}$

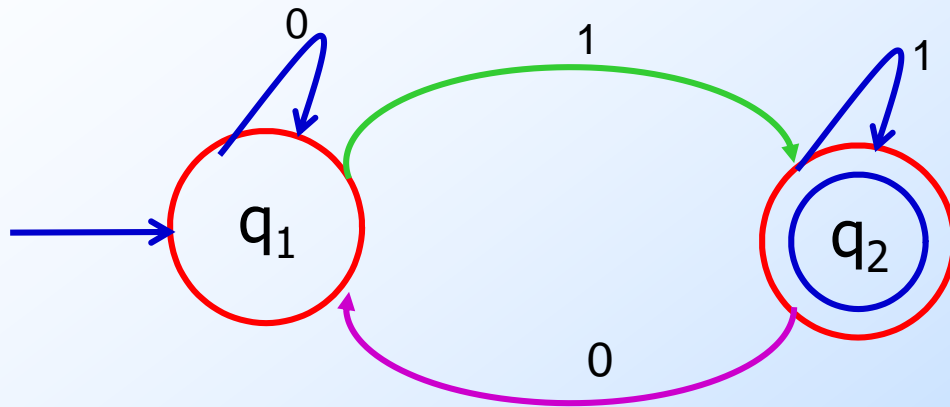
2. $\Sigma = \{0, 1\}$

3. δ try yourself

4. Start state: q_1

5. Final state: $\{q_2\}$

Example



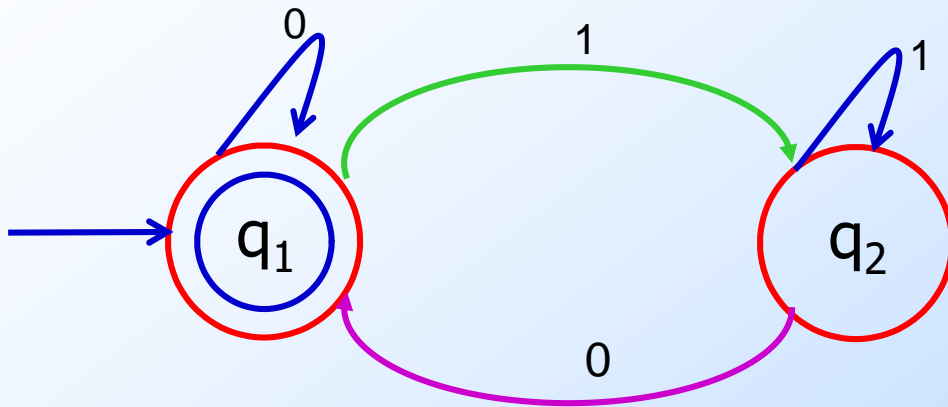
◆ $A_2 = (\{q_1, q_2\}, (0,1), \delta, q_1, \{q_2\})$

◆ Transition function, δ

Try: 1101, 11010, 0011010
 $L(A_2) = \{w \mid w \text{ ends in a } 1\}$

	0	1
$\rightarrow q_1$	q_1	q_2
* q_2	q_1	q_2

Example



◆ $A_3 = (\{q_1, q_2\}, (0,1), \delta, q_1, \{q_1\})$

◆ Transition function, δ

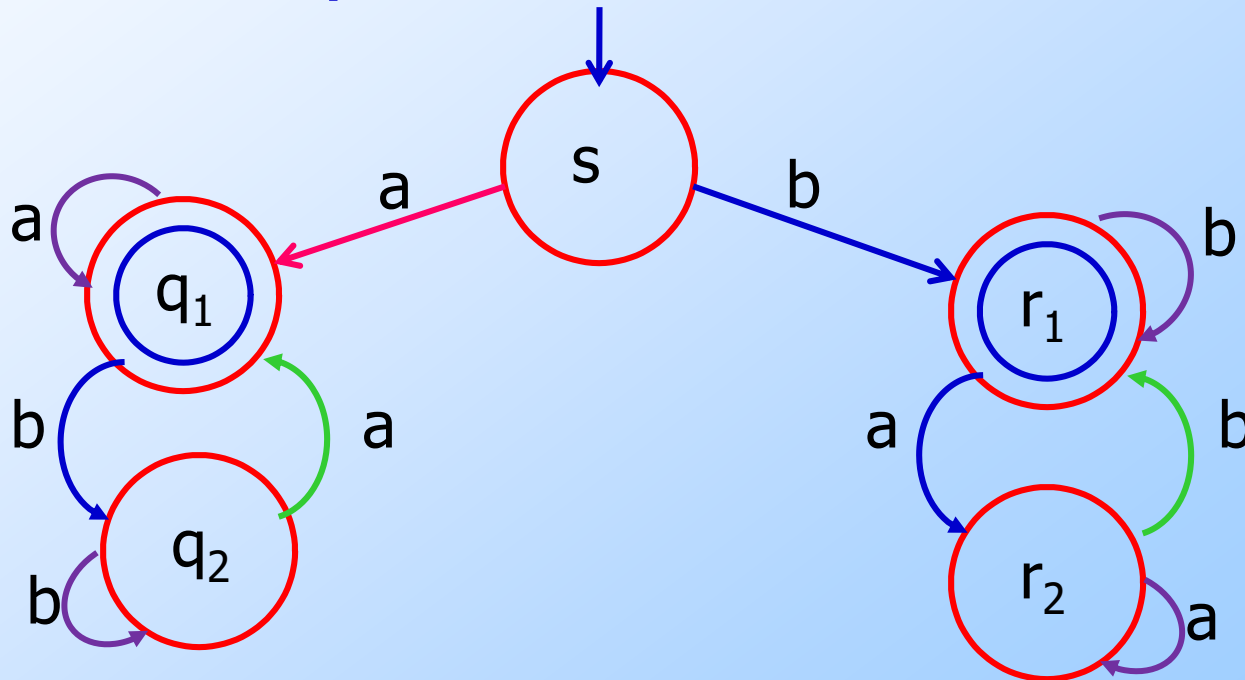
Try: 1101, 11010, 0011010

$L(A_3) = \{w \mid w \text{ is } \varepsilon \text{ or ends in a } 0\}$

	0	1
$\rightarrow^* q_1$	q_1	q_2
q_2	q_1	q_2

Example

- ◆ A_4 accepts all strings that start and end with a or that start and end with b
- ◆ A_4 accepts strings, that start and end with the same symbol

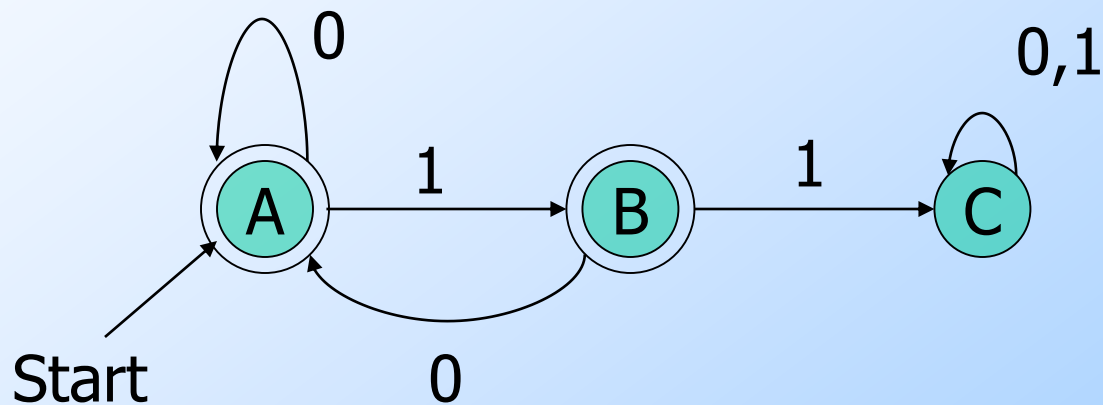


Language of a DFA

- ◆ Automata of all kinds define languages.
- ◆ If A is an automaton, $L(A)$ is its language.
- ◆ For a DFA A , $L(A)$ is the set of strings labeling paths from the start state to a final state.
- ◆ Formally: $L(A) =$ the set of strings w such that $\delta(q_0, w)$ is in F .

Example: String in a Language

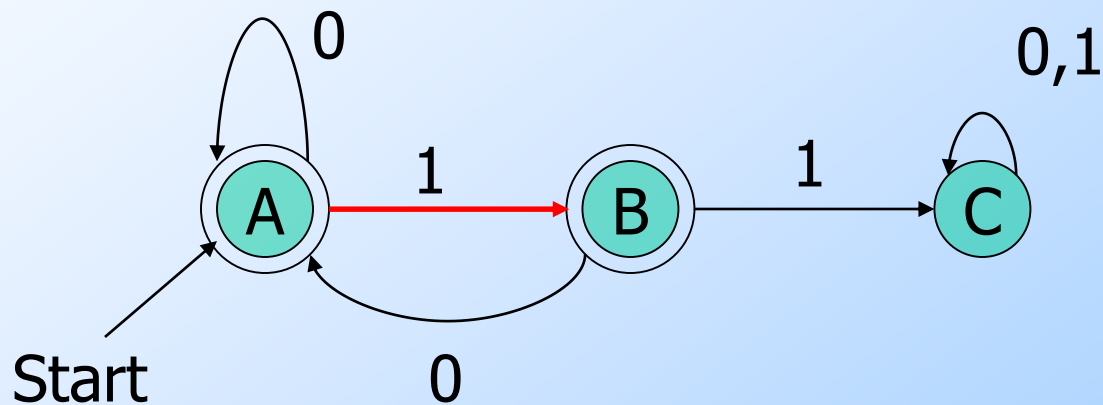
String 101 is in the language of the DFA below.
Start at A.



Example: String in a Language

String 101 is in the language of the DFA below.

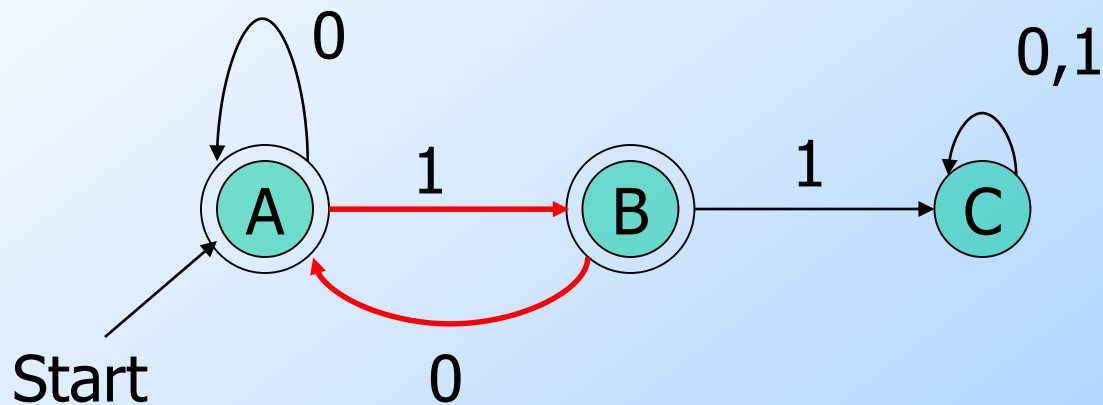
Follow arc labeled 1.



Example: String in a Language

String 101 is in the language of the DFA below.

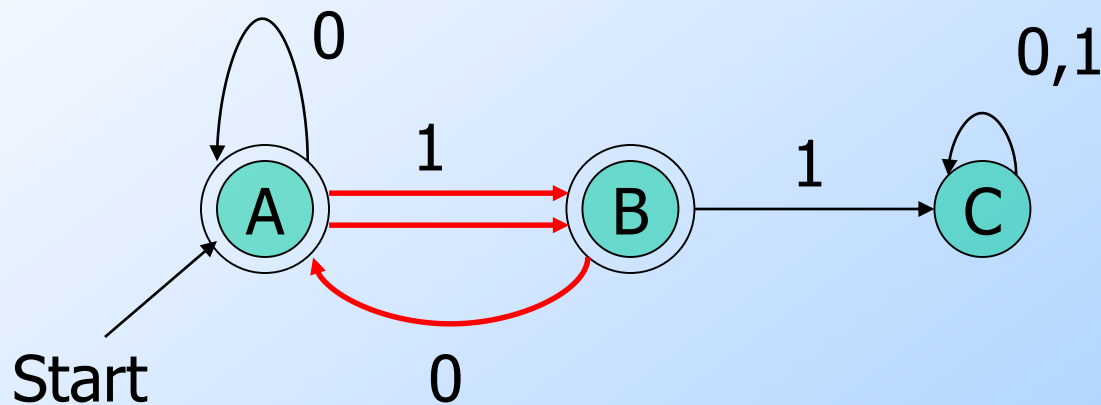
Then arc labeled 0 from current state B.



Example: String in a Language

String 101 is in the language of the DFA below.

Finally arc labeled 1 from current state A. Result is an accepting state, so 101 is in the language.



Example – Concluded

◆ The language of our example DFA is:
 $\{w \mid w \text{ is in } \{0,1\}^* \text{ and } w \text{ does not have two consecutive 1's}\}$

Such that...

These conditions about w are true.

Read a *set former* as
"The set of strings w ..."

Try Yourself

- ◆ Give DFA's accepting the following languages over the alphabet $\{0,1\}$
 - a) The set of all strings ending in 00
 - b) The set of all strings with three consecutive 0's (not necessarily at the end)
 - c) The set of strings with 011 as a substring

Try Yourself

- ◆ Give DFA's accepting the following languages over the alphabets $\{0,1\}$
 - a) The set of all strings such that each block of five consecutive symbols contains at least two 0's
 - b) The set of all strings whose tenth symbol from right end is a 1
 - c) The set of strings such that the number of 0's is divisible by five, and the number of 1's is divisible by 3